SN – 353

V Semester B.A./B.Sc. Examination, November/December 2017
(Semester Scheme) (CBCS) (2016 – 17 & Onwards)
(Fresh + Repeaters)
MATHEMATICS – V

Time : 3 Hours

Max. Marks : 70

***Instruction** : Answer **all** questions.*

PART – A

Answer **any five** questions : **(5×2=10)**

1 a) In a ring $(R, +, \cdot)$ prove that $\forall a, b, c \in R$, $a \cdot (b - c) = a \cdot b - a \cdot c$.

b) Show that the set of even integers is not an ideal of the ring of rational numbers.

c) Prove that every field is a principal ideal ring.

d) If $\vec{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$, show that $\vec{F}$ is irrotational.

e) Find the maximum directional derivative of $xsinz - ycosz$ at $(0, 0, 0)$.

f) Prove that $E\nabla = \nabla E = \Delta$.

g) Construct the Newton's divided difference table for the following data :

x	4	7	9	12
f(x)	– 43	83	327	1053

.

h) Using Trapezoidal rule to evaluate $\int_0^1 \frac{dx}{1+x}$ where

x	0	1/6	2/6	3/6	4/6	5/6	1
y = f(x)	1	0.8571	0.75	0.6667	0.6	0.5455	0.5

P.T.O.

7. a) Show that Curl $\left[\vec{r}\times(\vec{a}\times\vec{r})\right] = 3\vec{r}\times\vec{a}$ where $\vec{a}$ is constant vector and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

b) If the vector $\vec{F} = (3x + 3y + 4z)\,\hat{i} + (x - ay + 3z)\,\hat{j} + (3x + 2y - z)\,\hat{k}$ is solenoidal, find 'a'.

8. a) Prove that $\nabla^2\left(\frac{1}{r}\right) = 0$, where $r^2 = x^2 + y^2 + z^2$.

b) If $\vec{F} = \nabla(2x^3 y^2 z^4)$, find Curl $\vec{F}$ and hence verify that Curl $(\nabla\phi) = 0$.

OR

9. a) If ϕ is a scalar point function and $\vec{F}$ is a vector point function prove that

div $(\phi\vec{F})$ = ϕ div $\vec{F}$ + grad $\phi \cdot \vec{F}$

b) Find Curl (Curl $\vec{F}$) if $\vec{F} = x^2 y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$.

PART – D

(2×10=20)

Answer **two full** questions :

10. a) Use the method of separation of symbols to prove that

$u_0 + u_1 x + u_2 x^2 + \ldots$ to ∞

$= \frac{u_0}{1-x} + \frac{x\Delta u_0}{(1-x)^2} + \frac{x^2\Delta^2 u_0}{(1-x)^3} + \ldots$ to ∞.

b) i) Evaluate $\Delta^{10}[(1 - ax)(1 - bx^2)(1 - cx^3)(1 - dx^4)]$.

ii) Express $f(x) = 3x^3 + x^2 + x + 1$ as a factorial polynomial (taking h = 1).

OR

7. a) Show that Curl $\left[\vec{r}\times(\vec{a}\times\vec{r})\right] = 3\vec{r}\times\vec{a}$ where $\vec{a}$ is constant vector and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

   b) If the vector $\vec{F} = (3x + 3y + 4z)\,\hat{i} + (x - ay + 3z)\,\hat{j} + (3x + 2y - z)\,\hat{k}$ is solenoidal, find 'a'.

8. a) Prove that $\nabla^2\left(\frac{1}{r}\right) = 0$, where $r^2 = x^2 + y^2 + z^2$.

   b) If $\vec{F} = \nabla(2x^3y^2z^4)$, find Curl $\vec{F}$ and hence verify that Curl $(\nabla\phi) = 0$.

OR

9. a) If ϕ is a scalar point function and $\vec{F}$ is a vector point function, prove that

   div $(\phi\vec{F})$ = ϕ div $\vec{F}$ + grad $\phi \cdot \vec{F}$

   b) Find Curl (Curl $\vec{F}$) if $\vec{F} = x^2y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$.

PART – D

Answer **two full** questions : **(2×10=20)**

10. a) Use the method of separation of symbols to prove that

$$u_0 + u_1x + u_2x^2 + \ldots \text{ to } \infty$$

$$= \frac{u_0}{1-x} + \frac{x\Delta u_0}{(1-x)^2} + \frac{x^2\Delta^2 u_0}{(1-x)^3} + \ldots \text{ to } \infty.$$

   b) i) Evaluate $\Delta^{10}[(1 - ax)(1 - bx^2)(1 - cx^3)(1 - dx^4)]$.

   ii) Express $f(x) = 3x^3 + x^2 + x + 1$ as a factorial polynomial (taking $h = 1$).

OR

11. a) Find a second degree polynomial which takes the following data :

x	1	2	3	4
f(x)	–1	–1	1	5

b) Find f(1.9) from the following table :

x	1	1.4	1.8	2.2
f(x)	2.49	4.82	5.96	6.5

12. a) Using Lagrange's interpolation formula find f(6) for the following data :

x	2	5	7	10	12
f(x)	18	180	448	1210	2028

b) Using Simpson's $\frac{3}{8}^{th}$ rule evaluate $\int_0^{0.6} e^{-x^2}\,dx$ by taking 6 sub intervals.

OR

13. a) Following is the table of the normal weights of babies during the first few months of life.

Age in months	2	5	8	10	12
Weight in kgs	4.4	6.2	6.7	7.5	8.7

Estimate the weight of a baby of 7 months old using Newton's divided difference table.

b) Obtain an approximate value of $\int_0^6 \frac{dx}{1+x^2}$ by Simpson's $\frac{1}{3}^{rd}$ rule.